Course Structure and Introduction

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ECS at Southampton

- 117 Academic Staff

Research Groups

- Communications Signal Processing and Control.
- Electronics and Electrical Engineering.
- Nano.
- Optoelectronics Research Centre.
- Agents, Interaction and Complexity.
- Web and Internet Science.
- Electronic and Software Systems.
ECS Control Systems Academic Staff

- Professor Eric Rogers.
- Professor Paul Lewin.
- Professor Krzysztof Galkowski (Visiting Professor).
- Professor David H Owens (Visiting Professor).
- Dr Mark French (Reader).
- Dr Paolo Rapisarda (Senior Lecturer).
- Dr Ivan Markovsky (Senior Lecturer).
- Dr Chris Freeman (Lecturer).
- Dr Bing Chu (Lecturer).
Control Research Interests

- Nonlinear and Adaptive Control Theory/Applications [French].
- Behavioral Systems Theory [Rapisarda, French, Rogers].
- Identification [Markovsky, Rapisarda].
- Linear Systems Theory [Rapisarda, Markovsky].
- Flow Control [Rogers].
- Multidimensional (nD) Systems Theory/Repetitive Processes [Rogers, Galkowski, Owens].
- Iterative Learning Control Theory and Applications [Rogers, Freeman, Chu, Lewin, Owens, Galkowski, French].
- ILC in Stroke Rehabilitation [Freeman, Rogers].
- System Modeling and Identification in High Voltage Modeling and Control [Lewin, Rapisarda, Markovsky].
- Autonomous Underwater Vehicles [Rogers].
Control Lab Facilities in Southampton

- Multi-axis gantry robot and conveyor
  - Controller: Expansion card/C programming
    (4 input/ouput channels, 16 digital channels, 4 encoders)
- Legacy dispenser & conveyor
  - Controller: dSpace/C programming

(a) Gantry robot  (b) Legacy dispenser & conveyor
Control Lab Facilities in Southampton

- Non-minimum phase plant
- Controller: dSpace RTI 1104
  (4 input/output channels, 20 digital channels, 2 encoders)
Control Lab Facilities in Southampton

- Six degrees-of freedom pick and place robot.
Control Lab Facilities in Southampton

- Upper-limp Planar Rehabilitation workstation
Control Lab Facilities in Southampton

- Hocoma Rehabilitation Robot (2 off).
Course Structure

• Iterative learning control has been a recognized research area since the mid 1980’s and together with repetitive control remains a very active research area world-wide.

• This course will give an overview of the main areas of research with a particular focus on design algorithms that have been at least experimentally benchmarked.

• The aim is focus on the interface between the systems theory and applications and will include a recent application in stroke rehabilitation that represents a technology transfer between engineering and health sciences, a topic of increasing interest and driven by targeted research funding in the UK and elsewhere.
This course cont’d

• After motivation and introduction, the course will proceed to cover linear model based design methods, experimental benchmarking, stochastic iterative learning control, some advanced model based approaches and concludes with two case studies.

• It is very important to remember that parts of this course are influenced by the views and research interests of the presenter.

• It is also simply not possible to cover all topics and hence ‘pointers’ to further reading and study will also be given.
Iterative Learning Control (ILC) — Basics

• Motivation: Human Learning. Example: repeated training in sporting activities.
• Many processes are required to repeat the same operation over and over again but only over a finite duration. Examples: robot manipulators in assembly lines, chemical reactor processes, etc.
• Exact sequence: task is performed over the finite time interval \(0 \leq t \leq T < \infty\). Then the process or system resets to \(t = 0\) and the operation is repeated and so on.
• Notation and Terminology — each complete operation is called a trial (or a pass, or an iteration) and \(T < \infty\) is the trial duration (or length).
Iterative Learning Control (ILC) — Basics

- To specify a variable we need two co-ordinates:
- Trial index or number — subscript $k$
- Position or time along the trial — $t$ for continuous-time and $p$ for discrete.
- We write a variable as $y_k(t)$, $0 \leq t \leq T < \infty$, $k \geq 0$
Gantry Robot Testbeds

• Typical scenario: gantry robot undertaking a ‘pick and place’ operation where the following steps must be undertaken in synchronization with a conveyor system:
  • i) collect an object from a fixed location,
  • ii) transfer it over a finite duration,
  • iii) place it on a moving conveyor,
  • iv) return to the original location for the next object and then
  • v) repeat the previous four steps for as many objects as required.
Iterative Learning Control (ILC) — Basics

• If we apply the same control law on each trial we will get the same performance.

• Key idea in ILC — once the previous trial (or trials) have been (or are in the process of being) completed we can use previous (and current) trial outputs, inputs and errors to help improve performance sequentially from trial-to-trial.

• This is a 2D system/repetitive process — information propagation in two separate directions — from trial-to-trial \((k)\) and along a trial \((t)\).
Iterative Learning Control (ILC)— Early Literature

M. Uchyama
Formulation of high-speed motion pattern of a mechanical arm by trial

S. Arimoto, S. Kawamura and F. Miyazaki
Bettering Operation of Robots by Learning

D. A. Bristow, M. Tharayil and A. G. Alleyne
A Survey of Iterative Learning Control
Iterative Learning Control (ILC)— Early Literature

Hyo-Sung Ahn, YangQuan Chen and K. L. Moore

Iterative Learning Control: Brief Survey and Categorization

*IEEE Trans. on Systems, Man, and Cybernetics, Part-C,*

E. Rogers, K. Galkowski and D. H. Owens

*Control Systems Theory and Applications for Linear Repetitive Processes Series: Lecture Notes in Control and Information Sciences.*

D. H. Owens, E. Rogers, and P. L. Lewin

*Iterative Learning Control Algorithms and Experimental Benchmarking.*
A Sporting Analogy

(Courtesy of Doug Bristow et al)

• Consider a basketball player taking a free throw from a fixed location.
• Performance can be improved by repeated practice (trials).
• During each shot, the player observes the trajectory of the ball and consciously plans an alteration in the shooting motion for the next attempt.
• By continued practice, the correct motion is learned and becomes ingrained into the muscle memory and hence the shooting accuracy is iteratively improved.
• The converged muscle profile is an open-loop control generated through repetition (trials) and learning.
• This type of learned open-loop control strategy is at the heart of ILC.
ILC Versus Alternatives

• We consider systems that perform the same operation repeatedly and under the same operating conditions.

• In this case, a controller with no learning gives the same tracking error on each trial.

• Errors are repeated when trajectories are repeated!

• Each time the system is operated it will have the same overshoot, rise time, settling time and steady-state error.

• Error signals from previous trials are rich in information and so why not attempt to use this information!

• ILC aims to improve the transient response by adjusting the input to the system during future trials based on errors measured on the previous trial (or trials).
Iterative Learning Control (ILC)— Analysis

• Suppose that we require the system to produce a desired output, say $r(t)$, $0 \leq t \leq T < \infty$. Suppose also that $e_k(t) = r(t) - y_k(t)$ denotes the current trial error.

• Then the objective of constructing a sequence of control inputs such that the performance achieved is gradually improving with each successive pass can be refined to a convergence condition on the input and error, i.e.,

$$\lim_{k \to \infty} ||e_k|| = 0, \quad \lim_{k \to \infty} ||u_k - u_\infty|| = 0$$

• $|| \cdot ||$ is a signal norm in a suitably chosen function space — see below.

• $u_\infty$ is termed the learned control.
Iterative Learning Control (ILC)— Analysis

• $u_\infty(t)$ is termed the learned control and if ILC is successful then

$$r(t) = Gu_\infty(t)$$

so why not convert to the Laplace transform and solve for $U_\infty(s)$ and hence $u_\infty(t)$?

• **Answer:** Inverting the plant (directly) is often not a good idea.
History

Definition of ILC: first formally proposed by Arimoto et al. (1984)

\[
\begin{align*}
    u_{k+1}(t) &= u_k(t) + L \dot{e}_k(t) \\
    e_k(t) &= r(t) - y_k(t)
\end{align*}
\]

Key idea:
• Use previous trial (or trials) error information to update current trial control input.

Key features:
• Each direction is of finite duration \( t \in [0, T] \).
• The process is reset at the end of each trial.
• The initial conditions are reset before each new trial starts.
Classical-type ILC laws (Discrete)

D-type: \( u_{k+1}(p) = u_k(p) + K_k[e_k(p + 1) - e_k(p)] \)

P-type: \( u_{k+1}(p) = u_k(p) + K_k e_k(p + 1) \)

Phase-lead type: \( u_{k+1}(p) = u_k(p) + K_k e_k(p + \lambda) \)


Question: how does ILC differ from alternatives?

- **Other learning-type control algorithms:** Adaptive control (in all its versions and including neural network based approaches) modify the controller — a system — whereas ILC modifies the control input — a signal.
- In some applications, stroke rehabilitation, it is the input signal that must be very carefully controlled.
- ILC does have similarities with Repetitive Control (RC), where the latter is intended for continuous operation and the former for discontinuous operation.
- As noted above, ILC is suited to a gantry robot undertaking a pick and place task whereas an RC application is to control the read/write head on a hard disk drive.
ILC Versus Alternatives

- In the latter case, each trial is a complete rotation of the disk and the next trial immediately follows completion of the previous one.

- The core difference between ILC and RC is in the specification of the initial conditions at the start of each trial. In ILC the simplest case is when these are equal for each trial — although they could be different. In RC these are equal to the final conditions on the previous trial.

- In ILC there is always a time lapse between the end of one trial and the start of the next. This stoppage time can be used to advantage — zero-phase filtering of the error signal (covered later), in stroke rehabilitation rest time for the patient (covered later).

- RC is covered in more detail later in this section of the course.
ILC Versus Alternatives

Question: how does ILC compare with well designed feedback and feedforward control?

- ILC goal (basic case) — generate a feedforward control that tracks a specified reference or rejects a repeating disturbance.
- A feedback controller reacts to inputs and disturbances and hence there is always a lag in transient tracking.
- Feedforward control can eliminate this lag for known or measurable signals, such as the reference, but typically not for disturbances.
- ILC is anticipatory and can compensate for exogenous signals, such as repeating disturbances, in advance by learning using information from previous trials.
ILC Versus Alternatives

• ILC does not require that the exogenous signals — reference or disturbances — be known or measured. These signals must, however, repeat from trial-to-trial.

• A feedback controller can deal with variations or uncertainties in the system model — robust control — but a feedforward controller only works well to the extent that the system is ‘accurately known’. Unmodeled nonlinear behavior and disturbances can limit the effectiveness of feedforward control.

• ILC generates its open-loop control through iteration in the trial domain and this makes it robust to uncertainty.

• ILC cannot always provide perfect tracking — noise and non-repeating disturbances degrade performance.
ILC Versus Alternatives

- In common with feedback control, noise sensitivity can be limited by use of observers but only to the extent that the plant is known.
- As noted above, the trial-to-trial structure of ILC and, in particular, the reset between trials, allows for advanced filtering and signal processing. In particular, zero-phase filtering, which is non-causal, can be used for high-frequency attenuation without introducing phase lag.
- To reject non-repeating disturbances a feedback controller in combination with ILC is the best approach.
Repetitive Control (RC)

- RC can be used to track/reject arbitrary period signals of a fixed period.
- Tracking/Disturbance rejection of periodic signals is a requirement in many applications.
- Examples include Hard disk/CD drives, electrical power supply and robot motion.
- First application — power supply for a proton synchrotron.
- Control objective: Control the power supply curve (periodically) to a specified shape. Required precision — order of 0.1 V.
Repetitive Control (RC)

- Difficult to attain $10^{-4}$ precision by computing the inverse — identification problematic to this precision.
- Robust tracking against plant uncertainty also required.
- Alternatives needed.
- The first solution proposed was based on the fact that the reference signal is periodic and hence make the system learn the desired input by itself as follows.
  - Apply the reference signal to the plant.
  - Store the error signal for one period.
  - Feed the error back into the plant!
- **Question:** What is the link with ILC? We will return to this question later in the course.
Multi-axis Gantry Robot

X-axis:
A brushless linear dc motor and an un-powered linear bearing slide;

Y-axis:
a single brushless linear dc motor

Z-axis:
a linear ball-screw stage driven by a rotary brushless dc motor

Position feedback is obtained by means of optical incremental encoders.
X-Axis Modeling

All axes of the gantry robot can be modeled by means of frequency response tests, and a continuous-time transfer-function obtained. Below is the measured X axis frequency response and that of the fitted model.
• The transfer-function for the X-axis:

\[
G_x(s) = \frac{13077183.4436(s + 113.4)}{s(s^2 + 61.57s + 1.125 \times 10^4)(s^2 + 227.9s + 5.647 \times 10^4)} \times \frac{s^2 + 30.28s + 2.13 \times 10^4}{s^2 + 466.1s + 6.142 \times 10^5}
\]

(1)

• In practical implementation, a discrete model is obtained by sampling using, for example, the Zero Order Hold (ZOH) method with sampling period \(T_s = 0.01\) sec.
\( Y \) and \( Z \)-Axis Dynamics

- **\( Y \)-axis model \((j = \sqrt{-1})\)**

\[
G_y(s) = \frac{(s + 148.20)(s + 49.4 \pm j526.52)}{s(s + 78.54j533.34)(s + 213.42 \pm j151.47)}
\]

- **\( Z \)-axis model**

\[
G_z(s) = \frac{15.8869(s + 850.3)}{s(s^2 + 707.6s + 3.377 \times 10^5)}
\]
3D Reference Signal for the Gantry Robot

Reference trajectory

- z-axis (m)
- y-axis (m)
- x-axis (m)

pick
place
Non-minimum phase system

- This experimental test-bed has previously been used to evaluate a number of ILC and RC schemes and consists of a rotary mechanical system of inertias, dampers, torsional springs, a timing belt, pulleys and gears.

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Non-minimum phase system

\[\theta_i \rightarrow J_g \rightarrow K_r \rightarrow \theta_o \]

\[B_r \rightarrow J_r \rightarrow G_r\]
Non-minimum phase system

- The non-minimum phase characteristic is achieved by using the arrangement shown in the figure above where $\theta_i$ and $\theta_o$ are the input and output positions, $J_r$ and $J_g$ are inertias, $B_r$ is a damper, $K_r$ is a spring and $G_r$ represents the gearing.
- A further spring-mass-damper system is connected to the input in order to increase the relative degree and complexity of the system.
- A 1000 pulse/rev encoder records the output shaft position and a standard squirrel cage induction motor supplied by an inverter, operating in Variable Voltage Variable Frequency (VVFV) mode, drives the load. The control scheme has been implemented in DOS to increase the available sampling frequency.
Non-minimum phase system

- The system has been modeled using a least mean squares algorithm to fit a linear model to a great number of frequency response test results.
- The resulting continuous-time plant transfer-function is

\[ G(s) = \frac{1.202(4 - s)}{s(s + 9)(s^2 + 12s + 56.25)}. \]  

- A PID feedback control loop around the plant is used in order to act as a pre-stabilizer and provide greater stability. The PID gains used (proportional, integral and derivative) are \( K_p = 137, K_i = 5 \) and \( K_d = 3 \).
Mathematics Interlude

• We need the concepts of a contraction mapping and a fixed point. In the case of the latter, a mapping $T : X \rightarrow X$ of a set $X$ into itself a fixed point is mapped onto itself (or kept fixed by $T$) and hence $Tx = x$.

• The next step is to consider the iteration
  $x_{h+1} = Tx_h, \ h = 0, 1, 2, \ldots, \text{with } x_0 \text{ arbitrary.}$

Definition
Let $X = (X, d)$ be a metric space. A mapping $T : X \rightarrow X$ is called a contraction on $X$ if there is a positive real number $\gamma < 1$ such that for all $x, y \in X$

$$d(Tx, Ty) \leq \gamma d(x, y)$$
Mathematics Interlude

Definition
A metric space is a pair \((X, d)\) where \(X\) is the set and \(d\) the metric (distance function on \(X\)), i.e., a function on \(X \times X\) such that for all \(x, y, z \in X\) i) \(d\) is real-valued, finite and nonnegative, ii) \(d(x, y) = 0\) if and only if \(x = y\), iii) \(d(x, y) = d(y, x)\) (symmetry) and iv) \(d(x, y) \leq d(x, z) + d(z, y)\) (triangle inequality).

Theorem
Consider a metric space \((X, d)\) where \(X\) is not the empty set. Suppose also that \(X\) is complete (all convergent sequences converge to a point in the set) and let \(T : X \rightarrow X\) be a contraction on \(X\). Then \(T\) has precisely one fixed point.
Mathematics Interlude

- Under the conditions of this theorem the iterative sequence $x_{h+1} = T x_h$, $h = 0, 1, 2, \ldots$, with $x_0$ arbitrary converges to the unique fixed point of $X$. Also we have the error estimate

$$d(x_j, x) \leq \frac{\gamma^j}{1 - \gamma} d(x_0, x_1)$$

- A normed linear space is a vector space with a norm, $\| \cdot \|$, defined on it which has the properties i) $\| x \| \geq 0$, ii) $\| x \| = 0 \iff x = 0$, iii) $\| \beta x \| = |\beta| \| x \|$, iv) $\| x + y \| \leq \| x \| + \| y \|$ (triangle inequality).

- A norm on $X$ defines a metric $d$ on $X$ which is given by

$$d(x, y) = \| x - y \|$$
Mathematics Interlude

• A complete normed linear space is termed a Banach space.

• It is possible to have metric spaces that are not normed spaces.

• This functional analysis theory is extensively used in engineering-related tasks — such as solving systems of linear equations iteratively using, for example, Gauss-Seidel iteration.

• Also in control theory — stability etc.
Mathematics Interlude

• An inner product space is a vector space $X$ with an inner product defined on $X$.
• An inner product on $X$ is a mapping of $X \times X$ into the scalar field $K$ of $X$, i.e., with every pair of vectors $x$ and $y$ there is associated a scalar that is written as

$$\langle x, y \rangle$$

and termed the inner product of $x$ and $y$, such that for all vectors $x, y$ and $z$ and all scalars $\gamma$

• $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$
• $\langle \gamma x, y \rangle = \gamma \langle x, y \rangle$
• $\langle x, y \rangle = \overline{\langle y, x \rangle}$
Mathematics Interlude

- $\langle x, x \rangle \geq 0$
- $\langle x, x \rangle = 0 \iff x = 0$

An inner product on $X$ defines a norm on $X$ given by

$$||x|| = \sqrt{\langle x, x \rangle}$$

and a metric on $X$ given by

$$d(x, y) = ||x - y|| = \sqrt{\langle x - y, x - y \rangle}$$

A Hilbert space is a Banach space
Mathematics Interlude

• Let $T : H_1 \rightarrow T_2$ be a bounded linear operator (finite induced operator norm), where $H_1$ and $H_2$ are Hilbert spaces. Then the Hilbert adjoint operator, $T^*$, of $T$ is the operator

$$T^* : H_2 \rightarrow H_1$$

such that for all $x \in H_1$ and $y \in H_2$

$$\langle Tx, y \rangle = \langle x, T^*y \rangle$$

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E. Kreyszig

*Introductory Functional Analysis with Applications.*